17. Find the maximum and minimum values of y from the following table :

x :	0	1	2	3	4	5
$f(x)$:	0	0.25	0	2.25	16	56.25

18. Find the value of x for which $f(x)$ is minimum, using the table

x :	9	10	11	12	13	14
$f(x)$:	1330	1340	1320	1250	1120	930

Also find the maximum value of $f(x)$?

30.4 NUMERICAL INTEGRATION

Unit 4 start from here

The process of evaluating a definite integral from a set of tabulated values of the integrand $f(x)$ is called *numerical integration*. This process when applied to a function of a single variable, is known as *quadrature*.

The problem of numerical integration, like that of numerical differentiation, is solved by representing $f(x)$ by an interpolation formula and then integrating it between the given limits. In this way, we can derive quadrature formula for approximate integration of a function defined by a set of numerical values only.

30.5 NEWTON-COTES QUADRATURE FORMULA

Let $$I = \int_a^b f(x)\,dx$$

where $f(x)$ takes the values $y_0, y_1, y_2, \ldots y_n$ for $x = x_0, x_1, x_2, \ldots x_n$. (Fig. 30.1)

Let us divide the interval (a, b) into n sub-intervals of width h so that $x_0 = a$, $x_1 = x_0 + h$, $x_2 = x_0 + 2h, \ldots x_n = x_0 + nh = b$. Then

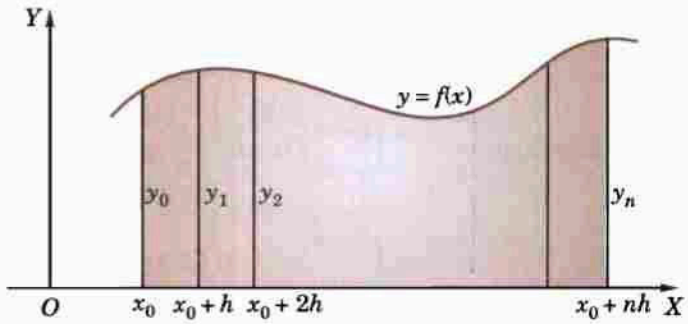

Fig. 30.1

$$I = \int_{x_0}^{x_0+nh} f(x)\,dx = h\int_0^n f(x_0 + rh)\,dr,$$

putting $x = x_0 + rh$, $dx = hdr$

$$= h\int_0^n \Big[y_0 + r\,\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 y_0 + \frac{r(r-1)(r-2)(r-3)(r-4)}{5!}\Delta^5 y_0 + \frac{r(r-1)(r-2)(r-3)(r-4)(r-5)}{6!}\Delta^6 y_0 + \cdots \Big] dr$$

[By Newton's forward interpolation formula]

Integrating term by, we obtain

$$\int_{x_0}^{x_0+nh} f(x)\,dx = nh\Big[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)^2}{24}\Delta^3 y_0 + \left(\frac{n^4}{5} - \frac{3n^3}{2} + \frac{11n^2}{3} - 3n\right)\frac{\Delta^4 y_0}{4!} + \left(\frac{n^5}{6} - 2n^4 + \frac{35n^3}{4} - \frac{50n^2}{3} + 12n\right)\frac{\Delta^5 y_0}{5!} + \left(\frac{n^6}{7} - \frac{15n^5}{6} + 17n^4 - \frac{225n^3}{4} + \frac{274n^2}{3} - 60n\right)\frac{\Delta^6 y_0}{6!} + \cdots \Big] \quad \text{...(A)}$$

This is known as *Newton-Cotes quadrature formula*. From this general formula, we deduce the following important quadrature rules by taking $n = 1, 2, 3 \ldots$

30.6 TRAPEZOIDAL RULE

Putting $n = 1$ in (A) § 30.5 and taking the curve through (x_0, y_0) and (x_1, y_1) as a straight line *i.e.* a polynomial of first order so that differences of order higher than first become zero, we get

$$\int_{x_0}^{x_0+h} f(x)\,dx = h\left(y_0 + \frac{1}{2}\Delta y_0\right) = \frac{h}{2}(y_0 + y_1)$$

Similarly $$\int_{x_0}^{x_0+2h} f(x)\,dx = h\left(y_1 + \frac{1}{2}\Delta y_1\right) = \frac{h}{2}(y_1 + y_2)$$

....

$$\int_{x_0+(n-1)h}^{x_0+nh} f(x)\,dx = \frac{h}{2}(y_{(n-1)} + y_n)$$

Adding these n integrals, we obtain

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})]$$

This is known as the ***trapezium rule.***

Obs. The area of each strip (trapezium) is found separately. Then the area under the curve and the *ordinates* at x_0 and $x_0 + nh$ is approximately equal to the areas of the trapeziums.

30.7 SIMPSON'S ONE-THIRD RULE

Putting $n = 2$ in (A) above and taking the curve through (x_0, y_0), (x_1, y_1) and (x_2, y_2) as a parabola *i.e.*, a polynomial of second order so that differences of order higher than second vanish, we get

$$\int_{x_0}^{x_0+2h} f(x)\,dx = 2h\left(y_0 + \Delta y_0 + \frac{1}{6}\Delta^2 y_0\right)\frac{h}{3}(y_0 + 4y_1 + y_2)$$

Similarly, $$\int_{x_0+2h}^{x_0+nh} f(x)\,dx = \frac{h}{3}(y_2 + 4y_3 + y_4)$$ when

....

$$\int_{x_0+(n-2)h}^{x_0+nh} f(x)\,dx = \frac{h}{3}(y_{n-2} + 4y_{n-1} + y_n),$$ n being even.

Adding all these integrals, we have (when n is even)

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3) + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$$

This is known as the *Simpson's one-third rule* or simply *Simpson's rule* and is most commonly used.

Obs. While applying *Simpson's 1/3rd rule, the given interval must be divided into even number of equal subintervals,* since we find the area of two strips at a time.

30.8 SIMPSON'S THREE-EIGHTH RULE

Putting $n = 3$ in (A) above and taking the curve through (x_i, y_i): $i = 0, 1, 2, 3$ as a polynomial of third order so that differences above the third order vanish, we get

$$\int_{x_0}^{x_0+3h} f(x)\,dx = 3h\left(y_0 + \frac{3}{2}\Delta y_0 + \frac{3}{4}\Delta^2 y_0 + \frac{1}{8}\Delta^3 y_0\right)$$

$$= \frac{3h}{8}(y_0 + 3y_1 + 3y_2 + y_3)$$

Similarly,

$$\int_{x_0+3h}^{x_0+nh} f(x)\,dx = \frac{3h}{8}(y_3 + 3y_4 + 3y_5 + y_6)$$ and so on.

Adding all such expressions from x_0 to $x_0 + nh$, where n is a multiple of 3, we obtain

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \dots + y_{n-1}) + 2(y_3 + y_6 + \dots + y_{n-3}]$$

which is known as *Simpson's three-eighth rule.*

Obs. While applying *Simpson's 3/8th rule, the number of sub-intervals should be taken as multiple of 3.*

30.9 BOOLE'S RULE

This method is not in the syallbus

Putting $n = 4$ in (A) above and neglecting all differences above the fourth, we obtain

$$\int_{x_0}^{x_0+4h} f(x)\,dx = 4h\left(y_0 + 2\Delta y_0 \frac{5}{3}\Delta^2 y_0 + \frac{2}{3}\Delta^3 y_0 + \frac{7}{90}\Delta^4 y_0\right)$$

$$= \frac{2h}{45}(7y_0 + 32y_1 + 12y_2 + 32y_3 + 7y_8)$$

Similarly

$$\int_{x_0+4h}^{x_0+8h} f(x)\,dx = \frac{2h}{45}(7y_4 + 32y_5 + 12y_6 + 32y_7 + 7y_8) \text{ and so on.}$$

Adding all these integrals from x_0 to $x_0 + nh$, where n is a multiple of 4, we get

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{2h}{45}(7y_0 + 32y_1 + 12y_2 + 32y_3 + 14y_4 + 32y_5 + 12y_6 + 32y_7 + 14y_8 + \ldots$$

This is known as *Boole's rule*.

Obs. While applying *Boole's rule*, the number of sub-intervals should be taken as a multiple of 4.

30.10 WEDDLE'S RULE

This method is not in the syallbus

Putting $n = 6$ in (A) above and neglecting all differences above the sixth, we obtain

$$\int_{x_0}^{x_0+6h} f(x)\,dx = \left(y_0 + 3\Delta y_0 + \frac{9}{2}\Delta^2 y_0 + 4\Delta^3 y_0 + \frac{123}{60}\Delta^4 y_0 + \frac{11}{20}\Delta^5 x_0 + \frac{1}{6}\cdot\frac{41}{140}\Delta^6 y_0\right)$$

If we replace $\frac{41}{140}\Delta^6 y_0$ by $\frac{3}{10}\Delta^6 y_0$, the error made will be negligible.

$$\therefore \quad \int_{x_0}^{x_0+6h} f(x)\,dx = \frac{3h}{10}(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6)$$

Similarly

$$\int_{x_0+6h}^{x_0+12h} f(x)\,dx = \frac{3h}{10}(y_6 + 5y_7 + y_8 + 6y_9 + y_{10} + 5y_{11} + y_{12}) \text{ and so on.}$$

Adding all these integrals from x_0 to $x_0 + nh$, where n is a multiple of 6, we get

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{10}(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + 2y_6 + 5y_7 + y_8 + \ldots)$$

This is known as *Weddle's rule*.

Obs. While applying *Weddle's rule the number of sub-intervals should be taken as a multiple of 6*. Weddle's rule is generally more accurate than any of the others. Of the two Simpson rules, the 1/3 rule is better.

Example 30.7. *Evaluate* $\int_0^6 \frac{dx}{1+x^2}$ *by using (i) Trapezoidal rule,*

(i) Simpson's 1/3 rule, *(Mumbai, 2005)*
(ii) Simpson's 3/8 rule, *(J.N.T.U., 2008)*
(iii) Weddle's rule and compare the results with its actual value. *(V.T.U., 2008)*

Solution. Divide the interval (0, 6) into six parts each of width $h = 1$. The values of $f(x) = \frac{1}{1+x^2}$ are given below :

x	0	1	2	3	4	5	6
$f(x)$	1	0.5	0.2	0.1	0.05884	0.0385	0.027
$= y$	y_0	y_1	y_2	y_3	y_4	y_5	y_6

(*i*) By Trapezoidal rule,

$$\int_0^6 \frac{1}{1+x^2} = \frac{h}{2}[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)]$$

$$= \frac{1}{2}[(1+\ 0.027)+2(0.5+0.2+0.1+0.0588+0.0385)] = 1.4108.$$

(*ii*) By Simpson's 1/3 rule,

$$\int_0^6 \frac{1}{1+x^2} = \frac{h}{3}[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)]$$

$$= \frac{1}{3}[(1+0.027)+4(0.5+0.1+0.0385)+2(0.2+0.0588)] = 1.3662.$$

(*iii*) By Simpson's 3/8 rule,

$$\int_0^6 \frac{1}{1+x^2} = \frac{3h}{8}[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2y_3]$$

$$= \frac{3}{8}[(1+0.027)+3(0.5+0.2+0.0588+0.0385)+2(0.1)] = 1.3571.$$

(*iv*) By Weddle's rule,

$$\int_0^6 \frac{1}{1+x^2} = \frac{3h}{10}[y_0+5y_1+y_2+6y_3+y_4\ 5y_5+y_6]$$

$$= 0.3[1+5(0.5)+0.2+6(0.1)+0.0588+5(0.0385)+0.027] = 1.3735.$$

Also, $\int_0^6 \frac{dx}{1+x^2} = \left|\tan^{-1} x\right|_0^6 = 1.4056$

Obs. This shows that the value of the integral found by Weddle's rule is the nearest to the actual value followed by its value given by Simpson's 1/3rd.

Example 30.8. *Use the Trapezoidal rule to estimate the integral* $\int_0^2 e^{x^2}\,dx$ *taking 10 intervals.*

(*U.P.T.U., 2008*)

Solution. Let $y = e^{x^2}$, $h = 0.2$ and $n = 10$.

The values of x and y are as follows :

x :	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
y :	1	1.0408	1.1735	1.4333	1.8964	2.1782	4.2206	7.0993	12.9358	25.5337	54.5981
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

By Trapezoidal rule, we have

$$\int_0^1 e^{x^2}dx = \frac{h}{2}[(y_0+y_{10})+2(y_1+y_2+y_3+y_4+y_5+y_6+y_7+y_8+y_9)]$$

$$= \frac{0.2}{2}[(1+54.5981)+2(1.0408+1.1735+1.4333+1.8964$$
$$+2.178+4.2206+7.0993+12.9358+25.5337)]$$

Hence $\int_0^2 e^{x^2}\,dx = 17.0621.$

Example 30.9. *Use Simpson's 1/3rd rule to find* $\int_0^{0.6} e^{-x^2}\,dx$ *by taking seven ordinates.*

(*V.T.U., 2011 ; Bhopal, 2009*)

Solution. Divide the interval (0, 0.6) into six parts each of width $h = 0.1$. The values of $y = f(x) = e^{-x^2}$ are given below :

x	0	0.1	0.2	0.3	0.4	0.5	0.6
x^2	0	0.01	0.04	0.09	0.16	0.25	0.36
y	1	0.9900	0.9608	0.9139	0.8521	0.7788	0.6977
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By Simpson's 1/3rd rule, we have

$$\int_0^{0.6} e^{-x^2} dx = \frac{h}{3} [(y_0 + y_6) + 4(y_1 + y_3 + y_5 + 2(y_2 + y_4)]$$

$$= \frac{0.1}{3} [(1 + 0.6977) + 4(0.99 + 0.9139 + 0.7788 + 2(0.9608 + 0.8521)]$$

$$= \frac{0.1}{3} [1.6977 + 10.7308 + 3.6258] = \frac{0.1}{3} (16.0543) = 0.5351.$$

Example 30.10. *Compute the value of* $\int_{0.2}^{1.4} (\sin x - \log x + e^x)\, dx$ *using Simpson's* $\frac{3}{8}$*th rule.*

(Mumbai, 2005)

Solution. Let $y = \sin x - \log_e x + e^x$ and $h = 0.2, n = 6$.

The values of y are as given below :

x :	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y :	3.0295	2.7975	2.8976	3.1660	3.5597	4.0698	4.4042
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By Simpson's $\frac{3}{8}$ th rule, we have

$$\int_{0.2}^{1.4} y\, dx = \frac{3h}{8} [(y_0 + y_6) + 2(y_3) + 3(y_1 + y_2 + y_4 + y_5)]$$

$$= \frac{3}{8} (0.2) [7.7336 + 2(3.1660) + 3 (13.3247)] = 4.053$$

Hence $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\, dx = 4.053.$

Obs. Applications of Simpson's rule. If the various ordinates in §30.5 represent equispaced cross-sectional areas, then Simpson's rule gives the volume of the solid. As such, Simpson's rule is very useful to civil engineers for calculating the amount of earth that must be moved to fill a depression or make a dam. Similar if the ordinates denote velocities at equal intervals of time, the Simpson's rule gives the distance travelled. The following examples illustrate these applications.

Example 30.11. *The velocity v (km/min) of a moped which starts from rests, is given at fixed intervals of time t (min) as follows :*

t :	2	4	6	8	10	12	14	16	18	20
y :	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes.

Solution. If s km be the distance covered in t (min), then $\frac{ds}{dt} = v$

$$\therefore \qquad \left| s \right|_{t=0}^{20} = \int_0^{20} v\, dt = \frac{h}{3} [X + 4.O + 2E], \text{ by Simpson's rule}$$

Hence $h = 2, v_0 = 0, v_1 = 10, v_2 = 18, v_3 = 25$ etc.

$$\therefore \qquad X = v_0 + v_{10} = 0 + 0 = 0$$

$$O = v_1 + v_3 + v_5 + v_7 + v_9 = 10 + 25 + 32 + 11 + 2 = 80$$
$$E = v_2 + v_4 + v_6 + v_8 = 18 + 29 + 20 + 5 = 72$$

Hence the required distance $= \left| s \right|_{t=0}^{20} = \frac{2}{3}(0 + 4 \times 80 + 2 \times 72)$

$= 309.33$ km.

Example 30.12. *The velocity v of a particle at distance s from a point on its linear path is given by the following table :*

s (m) :	*0*	*2.5*	*5.0*	*7.5*	*10.0*	*12.5*	*15.0*	*17.5*	*20.0*
v (m/sec) :	*16*	*19*	*21*	*22*	*20*	*17*	*13*	*11*	*9*

Estimate the time taken by the particle to traverse the distance of 20 metres, using Boole's rule.

) *(U.P.T.U. 2007)*

Solution. If t sec be the time taken to traverse a distance s (m) then $\frac{ds}{dt} = v$

or $\frac{dt}{ds} = \frac{1}{v} = y$ (say),

$\therefore$ then $\left| t \right|_{s=0}^{s=20} = \int_0^{20} y ds$

Here $h = 2.5$ and $n = 8$

Also $y_0 = \frac{1}{16}, y_1 = \frac{1}{19}, y_2 = \frac{1}{4}, y_3 = \frac{1}{22}, y_4 = \frac{1}{20}, y_5 = \frac{1}{17}, y_6 = \frac{1}{13}, y_7 = \frac{1}{11}, y_8 = \frac{1}{9}$

$\therefore$ by Boole's Rules, we have

$$\left| t \right|_{s=0}^{s=20} = \int_0^{20} y ds = \frac{2h}{45}\,[7y_0 + 32y_1 + 12y_2 + 32y_3 + 14y_4 + 32y_5 + 12y_6 + 32y_7 + 14y_8]$$

$$= \frac{2(2.5)}{45}\left[7\left(\frac{1}{16}\right) + 32\left(\frac{1}{19}\right) + 12\left(\frac{1}{21}\right) + 32\left(\frac{1}{22}\right) + 14\left(\frac{1}{20}\right) + 32\left(\frac{1}{17}\right) + 12\left(\frac{1}{13}\right) + 32\left(\frac{1}{11}\right) + 14\left(\frac{1}{9}\right)\right]$$

$$= \frac{1}{9}(12.11776) = 1.35$$

Hence the required time = 1.35 sec.

Example 30.13. *A solid of revolution is formed by rotating about the x-axis, the area between the x-axi the lines x = 0 and x = 1 and a curve through the points with the following co-ordinates*

x :	*0.00*	*0.25*	*0.50*	*0.75*	*1.00*
y :	*1.0000*	*0.9896*	*0.9589*	*0.9089*	*0.8415*

Estimate the volume of the solid formed using Simpson's rule. *(Raipur, 200*

Solution. Here $h = 0.25$, $y_0 = 1$, $y_1 = 0.9896$, $y_2 = 0.9589$, etc.

$\therefore$ Required volume of the solid generated

$$= \int_0^1 \pi y^2\, dx = \pi \cdot \frac{h}{3}\,[(y_0^2 + y_4^2) + 4(y_1^2 + y_3^2) + 2y_2^2]$$

$$= 0.25\,\frac{\pi}{3}\,[\{1 + (0.8415)^2\} + 4\{(0.9896)^2 + (0.9089)^2\} + 2(0.0589)^2]$$

$$= \frac{0.25 \times 3.1416}{3}\,[1.7081 + 7.2216 + 1.839] = 0.2618\,(10.7687) = 2.8192.$$

PROBLEMS 30.2

1. Evaluate $\int_0^1 \frac{dx}{1+x}$ applying

   (*i*) Trapezoidal rule (*J.N.T.U., 2009*) (*ii*) Simpson's 1/3rd rule

   (*iii*) Simpson's 3/8th rule. (*Mumbai, 2004*)

2. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using (*i*) Trapezoidal rule taking $h = 1/4$

   (*ii*) Simpson's 1/3rd rule taking $h = 1/4$. (*J.N.T.U., 2008*)

   (*iii*) Simpson's 3/8th rule taking $h = 1/6$. (*U.P.T.U, 2010 ; V.T.U., 2007*)

   (*iv*) Weddle's rule taking $h = 1/6$. (*Bhopal, 2009*)

   Hence compute an approximate value of π in each case.

3. Find an approximate value of $\log_e 5$ by calculating to 4 decimal places, by Simpson's 1/3 rule, $\int_0^5 \frac{dx}{4x+5}$, dividing the range into 10 equal parts. (*Anna., 2005*)

4. Evaluate $\int_0^6 x \sec x \, dx$ using eight intervals by Trapezoidal rule. (*U.P.T.U., 2009*)

5. Evaluate using Simpson's $\frac{1}{3}$rd rule (*i*) $\int_0^6 \frac{e^x}{1+x} dx$ (*U.P.T.U., 2006*)

   (*ii*) $\int_0^2 e^{-x^2} dx$ (Take $h = 0.25$). (*J.N.T.U., 2007*)

6. Evaluate using Simpson's 1/3rd rule $\int_0^1 \frac{dx}{x^3+x+1}$, choose step length 0.25. (*U.P.T.U., 2009*)

7. Evaluate using Simpson's 1/3rd rule, (*i*) $\int_0^\pi \sin x \, dx$ using 11 ordinates.

   (*ii*) $\int_0^{\pi/2} \sqrt{\cos\theta} \, d\theta$ taking 9 ordinates. (*V.T.U., 2009*)

8. Evaluate correct to 4 decimal places, by Simpson's $\frac{3}{8}$ th rule

   (*i*) $\int_0^9 \frac{dx}{1+x^3}$ (*U.P.T.U., M. Tech., 2010*) (*ii*) $\int_0^{\pi/2} e^{\sin x} dx$ (*U.P.T.U., 2007*)

9. Given that

x :	4.0	4.2	4.4	4.6	4.8	5.0	5.2
$\log x$:	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

   evaluate $\int^{5.2} \log x$ [illegible] by

   (*a*) Trape[illegible]idal r[illegible] (*b*) Simpson's 1/3rd rule, (*Kerala, 2003*)

   (*c*) Simp[illegible] 3/8 [illegible] T.U., 2006) (*d*) Weddle's rule (*V.T.U., 2008*)

10. Use Boo[illegible] compute $\int_0^{\pi/2} \sqrt{\sin x} \, dx$. (*U.P.T.U., 2008*)

11. The [illegible] (t) as a function of time :

t	[illegible]	5	6	7
$f(t)$	[illegible]	78	70	60

   Using Sim[illegible] (*J.N.T.U., 2007*)

12. A curve i[illegible] the following table :

x :	[illegible]	3.5	4
y :	[illegible]	2.6	2.1

   Estimate the area bounded by the cu[illegible] = 4. (*Bhopal, 2007*)

13. A river is 80 ft wide. The depth d in feet at a distance x ft. from one bank is given by the following table :

x :	0	10	20	30	40	50	60	70	80
y :	0	4	7	9	12	15	14	8	3

Find approximately the area of the cross-section. *(Rohtak, 2005)*

14. A curve is drawn to pass through the points given by following table :

x :	1	1.5	2	2.5	3	3.5	4
y :	2	2.4	2.7	2.8	3	2.6	2.1

Using Weddle's rule, estimate the area bounded by the curve, the x-axis and the lines $x = 1, x = 4$. *(V.T.U., 2011 S)*

15. A body is in the form of a solid of revolution. The diameter D in cms of its sections at distances x cm. from the one end are given below. Estimate the volume of the solid.

x :	0	2.5	5.0	7.5	10.0	12.5	15.0
D :	5	5.5	6.0	6.75	6.25	5.5	4.0

16. The velocity v of a particle at distances s from a point on its path is given by the table :

s ft :	0	10	20	30	40	50	60
v ft/sec :	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft. by using Simpson's 1/3 rule. *(U.P.T.U., 2007)*
Compare the result with Simpson's 3/8 rule. *(Madras, 2003)*

17. The following table gives the velocity v of a particle at time t :

t (second) :	0	2	4	6	8	10	12
v (m/sec) :	4	6	16	34	60	94	136

Find the distance moved by the particle in 12 seconds and also the acceleration at $t = 2$ sec. *(S.V.T.U., 2007)*

18. A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson's $\frac{1}{3}$rd rule, find the velocity of the rocket at $t = 80$ seconds.

t sec :	0	10	20	30	40	50	60	70	80
f (cm/sec^2) :	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

(Mumbai, 2004)

19. A reservoir discharging water through sluices at a depth h below the water surface has a surface area A for various values of h as given below :

h (ft.) :	10	11	12	13	14
A (sq.ft.) :	950	1070	1200	1350	1530

If t denotes time in minutes, the rate of fall of the surface is given by $dh/dt = -48\sqrt{h}/A$.
Estimate the time taken for the water level to fall from 14 to 10 ft. above the sluices.

30.12 OBJECTIVE TYPE OF QUESTIONS

PROBLEMS 30.3

Select the correct answer or fill up the blanks in the following questions :

1. The value of $\int_0^1 \frac{dx}{1+x}$ by Simpson's rule is

(*a*) 0.96315 (*b*) 0.63915 (*c*) 0.69315 (*d*) 0.69..

2. Using forward differences, the formula for $f'(a) = ...$

3. In application of Simpson's 1/3 rule, the interval h for closer approximation should be ...

4. $f(x)$ is given by

x :	0	0.5	1
$f(x)$:	1	0.8	0.5,

then using Trapezoidal rule, the value of $\int_0^1 f(x)\, dx$ is ...

5. If

x :	0	0.5	1	1.5	2
$f(x)$:	0	0.25	1	2.25	4

then the value of $\int_0^2 f(x)\,dx$ by Simpson's 1/3rd rule is ...

6. Simpson's 3/8th rule states that ...

7. For the data :

t :	3	6	9	12
$y(t)$:	–1	1	2	3,

the value of $\int_3^{12} y(t)\,dt$ when computed by Simpson's $\frac{1}{3}$rd rule is

(a) 15 (b) 10 (c) 0 (d) 5.

8. While evaluating a definite integral by Trapezoidal rule, the accuracy can be increased by taking ...

9. The value of $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's 1/3rd rule (taking $n = 1/4$) is ...

10. For the data:

x :	2	4	6	8
$f(x)$:	3	5	6	7,

$\int_2^8 f(x)\,dx$ when found by the Trapezoidal rule is

(a) 18 (b) 25 (c) 16 (d) 32.

11. The expression for $\left(\frac{dy}{dx}\right)_{x=x_0}$ using backward differences is ...

12. The number of strips required in Weddle's rule is ...

13. The number of strips required in Simpson's 3/8th rule is a multiple of

(a) 1 (b) 2 (c) 3 (d) 6.

14. If $y_0 = 1, y_1 = \frac{16}{17}, y_2 = \frac{4}{5}, y_3 = \frac{16}{25}, y_4 = \frac{1}{2}$ and $h = \frac{1}{4}$, then using Trapezoidal rule, $\int_0^4 ydx$ = ...

15. Using Simpson's $\frac{1}{3}$rd rule, $\int_0^1 \frac{dx}{x}$ = ... (taking $n = 4$).

16. If $y_0 = 1, y_1 = 0.5, y_2 = 0.2, y_3 = 0.1, y_4 = 0.06, y_5 = 0.04$ and $y_6 = 0.03$, then $\int_0^4 ydx$ by Simpson's $\frac{3}{8}$th, rule is = ...

17. If $f(0) = 1, f(1) = 2.7, f(2) = 7.4, f(3) = 20.1, f(4) = 54.6$ and $h = 1$, then $\int_0^4 f(x)\,dx$ by Simpson's $\frac{1}{3}$rd rule = ...

18. Simpson's 1/3rd rule and direct integration give the same result if ...

19. To evaluate $\int_{x_0}^{x_n} ydx$ by Simpson's 1/3rd rule as well as Simpson's 3/8th rule, the number of intervals should be .... and .... respectively.

20. Whenever Trapezoidal rule is applicable, Simpson's 1/3rd rule can also be applied. (True or False)