Histogram Features

- Histogram Features are based on the histogram of a region of the image.
- suppose u be a random variable which represents a gray level in a given region of the image.
- $\mathbf{P_u(\boldsymbol{x})} \triangleq \boldsymbol{Prob}[\boldsymbol{u} = \boldsymbol{x}] \approx \frac{\boldsymbol{no.of\ pixels\ with\ gray\ level\ x}}{\boldsymbol{total\ no.of\ pixels\ in\ the\ region}}$

Where x =0,1,2,3……………,L-1

Histogram Features

- Common features of Histogram are its :-

1. Mode
2. Mean
3. Median
4. Range
5. Variance
6. Standard Deviation
7. Co-Variance
8. Trace
9. Moments
10. Entropy

1. Mode

- Most common no. of a distribution i.e. Highest frequency

  eg.

- 1,2,2,2,2,2,2,3,4,5,6,7,8,8,8,9
- **Mode=2**

- **e.g.**
- Case 1st

  For odd numbers

  1,2,3,10,50

  Median=3

- **e.g.**
- Case 2nd

  For even numbers

  1,2,2,3,3,4

  Median=2,3

2. Mean

- Generally it is denoted by M.
- Mean=$\frac{Sum\ of\ the\ element}{total\ no.of\ element}$
- **M=**$\sum_{i=1}^{N} \frac{xi}{N}$ **where M=**mean

N=total no. of elements

x_i=sum of the total elements

eg.

1,1,2,3,4,4,8,8,8,8,8,9,9

N=13

Mean=$\frac{73}{13}$

Mean=5.6153

3. Median

- Middle No. Of a distribution when the numbers have been ordered i.e.. Sorted.

- Case 1^{st}

For odd numbers

median=$\left(\frac{N+1}{2}\right)^{th}$ element where N=total no of elements

- Case 2^{nd}

For even numbers

median=$\left(\frac{N}{2}\right)^{th}$ element **&** $\left(\frac{N}{2}\right)$**+1 element**

4. Range

- Difference between max & Min value of the data.

$$\textbf{Range}(x)\textbf{=max}(x)\textbf{-min}(x)$$

5. Variance

- Mean of squared deviations from the mean.
- **Var** $= \frac{\sum(\boldsymbol{xi} - \overline{\boldsymbol{x}})^2}{\boldsymbol{N}} = \frac{\sum \mathbf{d^2}}{\boldsymbol{N}}$
- **Var** $= \frac{\sum \mathbf{d^2}}{\boldsymbol{N}}$

$$\sigma^2 = \frac{mean\ \ squared\ deviation}{no.of\ observation}$$

where N=total no of element

σ^2 =variance

d=$(\boldsymbol{xi} - \overline{\boldsymbol{x}})$

$\boldsymbol{x}_i$=sum of the total elements

6. Standard Deviation

- Standard deviation is a square root of deviation.
- S.D.=$\sqrt{var} = \sqrt{\sigma^2} = \sigma$

$$\text{S.D.}(\sigma) = \sqrt{\frac{\sum(xi - \bar{x})^2}{N}}$$

7. Co-Variance

- Co-variance is very commonly used in statistical analysis as the basic for advanced statistics.

$$\mathbf{Cov\ (X,Y)} = E[(X-\mu_x)(Y-\mu_y)]$$

where E =expectation

$\mu = mean$

Cond…

- If the two variable are independent, the Co-variance is zero.
- If they are totally dependent the Co-variance of data can be arbitrarily large.
- The diagonal’s are the variance of each variable.
- If the row is an observation, each column a variable.

$$\textbf{Cov(X)}=\left(\frac{1}{N-1}\right)(X-mean(X)^T)\left(X-mean(X)\right)$$

where N= total no of elements

8. Trace

- Sum of the variance (sum of the elements of the diagonal of the Co-variance matrix)

9. Moments

- **Moments:** $m_i = \mathrm{E}\,[u^i] = \sum_{x=0}^{L-1} x^i\, p_u(x)$,

  where i=1,2,3,4……….

  where u is a random variable that represents a gray level

  m_i represents moments

  $p_u(x)$ represents histogram features

  u any random variable

- **Absolute moments:** $\widehat{m}_i = \mathrm{E}\,[|u|^i] = \sum_{x=0}^{L-1} |x|^i\, p_u(x)$

- **Central moments:** $\mu_i = \mathrm{E}\,\{[u - E(u)]^i\} = \sum_{x=0}^{L-1} (x - m_1)^i\, p_u(x)$

- **Absolute Central moments:** $\hat{\mu}_i = \mathrm{E}[|u - E(u)^i|] = \sum_{x=0}^{L-1} |x - m_1|^i\, p_u(x)$

Cond...

- m_1=mean or statistical average gray level
- m_2=Contrast or mean square value or Average energy
- $\hat{\mu}_1$=Dispersion
- σ^2= μ_2=variance
- μ_3=Skewness
- K=$\mu 4/\sigma^2$=Kurtosis= $\mu 4$-3
  ref:w.scribd.com/doc/28414452/9/Statistical-Moments-from-the-Grey-Level-Histograms
- u_{max}=Mode
- u_{med}=Median

10. Entropy

- **Entropy:** $H = E[-\log_2 p_u]$
  $= -\sum_{x=0}^{L-1} p_u(x) \log_2 p_u(x)$ bits

  Where $p_u(x)$ represents histogram features
  u=any random variable

- A narrow histogram indicates a low contrast region.
- Variance can be used to measure local activity in the amplitudes.
- Histogram features are also useful for shape analysis of objects from their projections.

Two-Dimensional Histogram

- Second-order joint probabilities or can say two-dimensional histogram is useful in feature extraction of textures applications.
- A second-order joint probability is defined as

$p_u(x_{1,} x_2) \triangleq p_{u1,u2}(x_{1,} x_2) \triangleq prob[u_{1=} x_1 , u_{2=} x_{2}],$
where x_1 , x_2 =0,……,L-1

$$\approx \frac{no.of\ pairs\ of\ pixels\ u1{=}x1, u2{=}x2}{total\ no.of\ such\ pairs\ of\ pixels\ in\ the\ region}$$

Where $p_u(x_{1,} x_2)$ represents two-dimensional histogram features

u1,u2 represents random variable

Cond.....

- suppose we have two pixels u_1 and u_2 at relative distance r *and orientation* θ,the distribution function can be written as

$$\mathbf{p_u}(x_1, x_2) = \mathbf{f}(r, \theta; x_1, x_2)$$

where θ =orientation

r=Relative distance

some useful texture features based on this function are ----

- Inertia: $I(r, \theta) \triangleq \sum_{x_1} \sum_{x_2} |x_1 - x_2|^2 \, f(r, \theta; X_1, X_2)$

- **M**eant distribution: $\mu(r; x_1, x_2) = \frac{1}{N_0} \sum_{\theta} f(r, \theta; x_1, x_2)$

  where N_0 represents the total no of orientations.

- **Variance distribution:**
  $\sigma^2(r; x_1, x_2) = \frac{1}{N_0} \sum_{\theta} [f(r, \theta; x_1, x_2) - \mu(r; x_1, x_2)]^2$

Cond…

Where $\boldsymbol{\sigma}^2$ **=**variance

$\boldsymbol{\mu}$= Mean distribution

Spread distribution

- Spread distribution:

$$\boldsymbol{\eta}((r; x_1, x_2) = \overset{max}{\theta} \{(\mathbf{f}(r, \theta; x_1, x_2))\} - \overset{min}{\theta} \{(\mathbf{f}(r, \theta; x_1, x_2))\}$$

where θ =orientation

r=Relative distance

N_0 represents the total no of orientations.

x_1, x_2 represents pixel values

References

1. Anil K. Jain, “Fundamentals of Digital Image Processing", PHI Learning Education ,Inc.,3rd Edition
2. **w.scribd.com/doc/28414452/9/Statistical-Moments-from-the-Grey-Level-Histograms**

Thank You!