THECONTROL SYSTEM

INTRODUCTION

In the previous chapters, the dynamic behavior of several basic systems was examined. With this background, we can extend the discussion to a complete control system and introduce the fundamental concept of feedback. In order to work with a familiar system, the treatment will be based on the illustrative example of Chap. 1, which is concerned with a stirred-tank heater.

Figure 9.1 is a sketch of the apparatus. To orient the reader, the physical description of this control system will be reviewed. A liquid stream at a temperature T_i enters an insulated, well-stirred tank at a constant flow rate w (mass/time). It is desired to maintain (or control) the temperature in the tank at T_R by means of the controller. If the measured tank temperature T_m differs from the desired temperature T_R, the controller senses the difference or ***error,*** $\epsilon = T_R - T_m$, and changes the heat input in such a way as to reduce the magnitude of ϵ. If the controller changes the heat input to the tank by an amount that is proportional to ϵ, ***we*** have ***proportional*** control.

In Fig. 9.1, it is indicated that the source of heat input q may be electricity or steam. If an electrical source were used, the final control element might be a variable transformer that is used to adjust current to a resistance.heating element; if steam were used, the final control element would be a control valve that adjusts the flow of steam. In either case, the output signal from the controller should adjust q in such a way as to maintain control of the temperature in the tank.

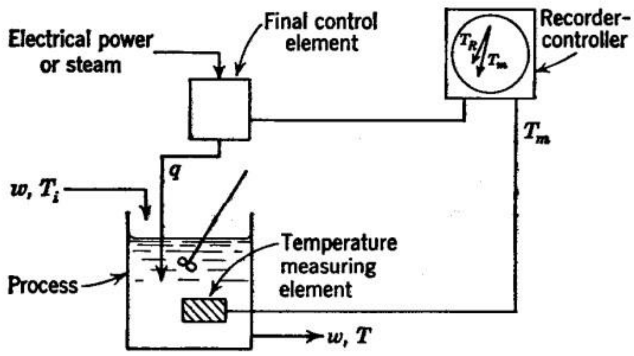

FIGURE 9-1
Control system for a stirred-tank heater.

Components of a Control System

The system shown in Fig. 9.1 may be divided into the following components:

1. Process (stirred-tank heater).
2. Measuring element (thermometer).
3. Controller.
4. Final control element (variable transformer or control valve).

Each of these components can be readily identified as a separate physical item in the process. In general, these four components will constitute most of the control systems that we shall consider in this text; however, the reader should realize that more complex control systems exist in which more components are used. For example, there are some processes which require a cascade control system in which two controllers and two measuring elements are used. A cascade system is discussed in Chap. 18.

Block Diagram

For computational purposes, it is convenient to represent the control system of Fig. 9.1 by means of the block diagram shown in Fig. 9.2. Such a diagram makes it much easier to visualize the relationships among the various signals. New terms, which appear in Fig. 9.2, are set ***point*** and ***loud.*** The set point is a synonym for the desired value of the controlled variable. The load refers to a change in any variable that may cause the controlled variable of the process to change. In this example, the inlet temperature T_i is a load variable. Other possible loads for this system are changes in flow rate and heat loss from the tank. (These loads are not shown on the diagram.)

The control system shown in Fig. 9.2 is called a ***closed-loop*** system or a feedback system because the measured value of the controlled variable is returned or "fed back" to a device called ***the comparator.*** In the comparator, the controlled variable is compared with the desired value or ***set point.*** If there is any difference

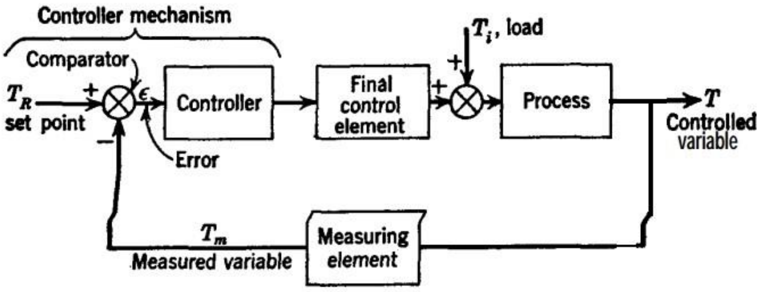

FIGURE 9-2
Block diagram of a simple control system.

between the measured variable and the set point, an error is generated. This error enters a ***controller,*** which in turn adjusts ***the final control element*** in order to return the controlled variable to the set point.

Negative Feedback versus Positive Feedback

Several terms have been used that may need further clarification. The feedback principle, which is illustrated by Fig. 9.2, involves the use of the controlled variable T to maintain itself at a desired value T_R. The arrangement of the apparatus of Fig. 9.2 is often described as ***negative feedback*** to contrast with another arrangement called positive feedback. Negative feedback ensures that the difference between T_R and T_m is used to adjust the control element so that the tendency is to reduce the error. For example, assume that the system is at steady state and that $T = T_m = T_R$. If the load T_i should increase, T and T_m would start to increase, which would cause the error ϵ to become negative. With proportional control, the decrease in error would cause the controller and final control element to ***decrease*** the flow of heat to the system with the result that the flow of heat would eventually be reduced to a value such that T approaches T_R. A verbal description of the operation of a feedback control system, such as the one just given, is admittedly inadequate, for this description necessarily is given as a sequence of events. Actually all the components operate simultaneously, and the only adequate description of what is occurring is a set of simultaneous differential equations. This more accurate description is the primary subject matter of the present and succeeding chapters.

If the signal to the comparator were obtained by adding T_R and ***T,, we*** would have a ***positive feedback*** system, which is inherently unstable. To see that this is true, again assume that the system is at steady state and that $T = T_m = T_R$. If T_i were to increase, T and T_m would increase, which would cause the signal the comparator (ϵ in Fig. 9.2) to increase, with the result that the heat to the system would increase. However, this action, which is just the opposite of that needed, would cause T to increase further. It should be clear that this situation would cause T to "run away" and control would not be achieved. For this reason, positive feedback would never be used intentionally in the system of Fig. 9.2. However, in more complex systems it may arise naturally. An example of this is discussed in Chap. 21.

Servo Problem versus Regulator Problem

The control system of Fig. 9.2 can be considered from the point of view of its ability to handle either of two types of situations. In the first situation, which is called the servomechanism-type (or servo) problem, we assume that there is no change in load T_i and that we are interested in changing the bath temperature according to some prescribed function of time. For this problem, the set point T_R would be changed in accordance with the desired variation in bath temperature. If the variation is sufficiently slow, the bath temperature may be expected to follow the variation in T_R very closely. There are occasions when a control system in the chemical industry will be operated in this manner. For example, one may be interested in varying the temperature of a reactor according to a prescribed time-temperature pattern. However, the majority of problems that may be described as the servo type come from fields other than the chemical industry. The tracking of missiles and aircraft and the automatic machining of intricate parts from a master pattern are well-known examples of the servo-type problem. The other situation will be referred to as the regulator problem. In this case, the desired value T_R is to remain fixed and the purpose of the control system is to maintain the controlled variable at T_R in spite of changes in load T_i. This problem is very common in the chemical industry, and a complicated industrial process will often have many self-contained control systems, each of which maintains a particular process variable at a desired value. These control systems are of the regulator type.

In considering control systems in the following chapters, we shall frequently discuss the response of a linear control system to a change in set point (servo problem) separately from the response to a change in load (regulator problem). However, it should be realized that this is done only for convenience. The basic approach to obtaining the response of either type is essentially the same, and the two responses may be superimposed to obtain the response to any linear combination of set-point and load changes.

DEVELOPMENT OF BLOCK DIAGRAM

Each block in Fig. 9.2 represents the functional relationship existing between the input and output of a particular component. In the previous chapters, such input-output relations were developed in the form of transfer functions. In block-diagram representations of control systems, the variables selected are deviation variables, and inside each block is placed the transfer function relating the input-output pair of variables. Finally, the blocks are combined to give the overall block diagram. This is the procedure to be followed in developing Fig. 9.2.

Process

Consider first the block for the process. This block will be seen to differ somewhat from those presented in previous chapters in that two input variables are present; however, the procedure for developing the transfer function remains the same.

An unsteady-state energy balance* around the tank gives

$$q + wC(T_i - T_o) - wC(T - T_o) = \rho C V \frac{dT}{dt} \tag{9.1}$$

where T_o is the reference temperature.

At steady state, dT/dt is zero, and Eq. (9.1) can be written

$$q_s + wC(T_{i_s} - T_o) - wC(T_s - T_o) = 0 \tag{9.2}$$

where the subscript s has been used to indicate steady state.

Subtracting Eq. (9.2) from Eq. (9.1) gives

$$q - q_s + wC[(T_i - T_{i_s}) - (T - T_s)] = \rho C V \frac{d(T - T_s)}{dt} \tag{9.3}$$

Notice that the reference temperature T_o cancels in the subtraction. If we introduce the deviation variables

$$T_i' = T_i - T_{i_s} \tag{9.4}$$

$$Q = q - q_s \tag{9.5}$$

$$T' = T - T, \tag{9.6}$$

Eq. (9.3) becomes

$$Q + wC(T_i' - T') = \rho C V \frac{dT'}{dt} \tag{9.7}$$

Taking the Laplace transform of Eq. (9.7) gives

$$Q(s) + wC[T_i'(s) - T'(s)] = \rho C V s T'(s) \tag{9.8}$$

or

$$T'(s)\left(\frac{\rho V}{w} s + 1\right) = \frac{Q(s)}{wC} + T_i'(s) \tag{9.9}$$

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*In this analysis, it is assumed that the flow rate of heat q is instantaneously available and independent of the temperature in the tank. In some stirred-tank heaters, such as a jacketed kettle, q depends on both the temperature of the fluid in the jacket and the temperature of the fluid in the kettle. In this introductory chapter, systems (electrically heated tank or direct steam-heated tank) are selected for which this complication can be ignored. In Chap. 21, the analysis of a steam-jacketed kettle is given in which the effect of kettle temperature on q is taken into account.

This last expression can be written

$$T'(s) = \frac{1/wC}{\tau s + 1}Q(s) + \frac{1}{\tau s + 1}T_i'(s) \tag{9.10}$$

where

$$\tau = \frac{\rho V}{w}$$

If there is a change in Q(t) only, then $T_i'(t) = 0$ and the transfer function relating T' to Q is

$$\frac{T'(s)}{Q(s)} = \frac{1/wC}{\tau s + 1} \tag{9.11}$$

If there is a change in $T_i'(t)$ only, then Q(t) = 0 and the transfer function relating T' to T_i' is

$$\frac{T'(s)}{T_i'(s)} = \frac{1}{\tau s + 1} \tag{9.12}$$

Equation (9.10) is represented by the block diagram shown in Fig. 9.3*a*. This diagram is simply an alternate way to express Eq. (9.10) in terms of the transfer functions of Eqs. (9.11) and (9.12). Superposition makes this representation possible. Notice that, in Fig. 9.3, we have indicated summation of signals by the symbol shown in Fig. 9.4, which is called a *summing junction.* Subtraction can also be indicated with this symbol by placing a minus sign at the appropriate input. The summing junction was used previously as the symbol for the comparator of the controller (see Fig. 9.2). This symbol, which is standard in the control literature, may have several inputs but only one output.

A block diagram that is equivalent to Fig. 9.3*a* is shown in Fig. 9.3*b*. That this diagram is correct can be seen by rearranging Eq. (9.10); thus

$$T'(s) = [Q(s) + wCT_i'(s)]\frac{1/wC}{\tau s + 1} \tag{9.13}$$

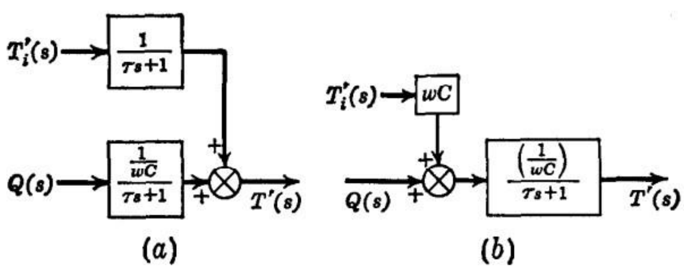

FIGURE 9-3
Block diagram for process.

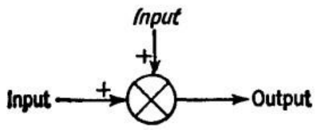

FIGURE 9-4
Summing junction.

In Fig. **9.3*b***, the input variables $Q(s)$ and $wCT_i'(s)$ **are** summed before being operated on by the transfer function $1/wC/(\tau s + 1)$.

The physical situation that exists for the control system (Fig. 9.1) if steam heating is used requires **more** careful analysis to show that Fig. 9.3 is an equivalent block diagram. Assume that a supply of steam at constant conditions is available for heating the **tank.** One method for introducing heat to the system is to let the steam flow through a control valve and discharge directly into the water in the tank, where it will condense completely and become part of the stream leaving the tank (see Fig. 9.5).

If the flow of steam, f (pounds/time), is small compared with the inlet flow w, the total outlet flow is approximately equal to w. When the system is at steady state, the heat balance may be written

$$wC(T_{i_s} - T_o) - wC(T_s - T_o) + f_s(H_g - H_{l_s}) = 0 \qquad (9.14)$$

where T_o = reference temperature used to evaluate enthalpy of all streams entering and leaving tank

H_g = specific enthalpy of the steam supplied, a constant

H_{l_s} = specific enthalpy of the condensed steam flowing out at T_s, as part of the total stream

The term H_{l_s} may be written in terms of heat capacity and temperature; thus

$$H_{l_s} = C(T_s - T_o) \qquad (9.15)$$

From this, we see that, if the steady-state temperature changes, H_{l_s} changes. In Eq. (**9.14**), $f_s(H_g - H_{l_s})$ is equivalent to the steady-state input q_s used previously, as **can** be seen by comparing Eq. (9.2) with (9.14).

Now consider an unsteady-state operation in whichfis much less than w and the temperature T of the bath does not deviate significantly from the steady-state

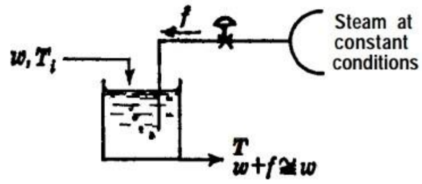

FIGURE 9-5
Supplying heat by steam.

temperature T_s. For these conditions, we may write the unsteady-state balance approximately; thus

$$wC(T_i - T_o) - wC(T - T_o) + f(H_g - H_{l_s}) = \rho C V \frac{dT}{dt} \tag{9.16}$$

In a practical situation for steam, H_g will be about 1000 Btu/lb$_m$. If the temperature of the bath, T, never deviates from T_s by more than $10°$, the error in using the term $f(H_g - H_{l_s})$ instead of $f(H_g - H_l)$ will be no more than 1 percent. Under these conditions, Eq. (9.16) represents the system closely, and by comparing Eq. (9.16) with Eq. (9.1), it is clear that

$$q = f(H_g - H_{l_s}) \tag{9.17}$$

Therefore, q is proportional to the flow of steam f, which may be varied by means of a control valve. It should be emphasized that the analysis presented here is only approximate. Both f and the deviation in T must be small. The smaller they become, the more closely Eq. (9.16) represents the actual physical system. An exact analysis of the problem leads to a differential equation with time-varying coefficients, and the transfer-function approach does not apply. The problem becomes considerably more difficult. A better approximation will be discussed in Chap. 21, where linearization techniques are used.

Measuring Element

The temperature-measuring element, which senses the bath temperature T and transmits a signal T_m to the controller, may exhibit some dynamic lag. From the discussion of the mercury thermometer in Chap. 5, we observed this lag to be first-order. In this example, we shall assume that the temperature-measuring element is a first-order system, for which the transfer function is

$$\frac{T'_m(s)}{T'(s)} = \frac{1}{\tau_m s + 1} \tag{9.18}$$

where the input-output variables T' and T'_m are deviation variables, defined as

$$T' = T - T_s$$

$$T'_m = T_m - T_{m_s}$$

Note that, when the control system is at steady state, $T_s = T_{m_s}$, which means that the temperature-measuring element reads the true bath temperature. The transfer function for the measuring element may be represented by the block diagram shown in Fig. 9.6.

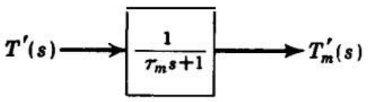

FIGURE 9-6
Block diagram of measuring element.

Controller and Final Control Element

For convenience, the blocks representing the controller and the final control element are combined into one block. In this way, we need be concerned only with the overall response between the error and the heat input to the tank. Also, it is assumed that the controller is a proportional controller. (In the next chapter, the response of other controllers, which are commonly used in control systems, will be described.) The relationship for a proportional controller is

$$q = K_c\epsilon + A \tag{9.19}$$

where $\epsilon = T_R - T_m$

T_R = set-point temperature

K_c = proportional sensitivity or controller gain

A = heat input when $\epsilon = 0$

At steady state, it is assumed* that the set point, the process temperature, and the measured temperature are all equal to each other; thus

$$T_{R_s} = T_s = T_{m_s} \tag{9.20}$$

Let ϵ' be the deviation variable for error; thus

$$\epsilon' = \epsilon - \epsilon_s \tag{9.21}$$

where $\epsilon_s = T_{R_s} - T_{m_s}$

Since $T_{R_s} = T_{m_s}$, $\epsilon_s = 0$ and Eq. (9.21 becomes

$$\epsilon' = \epsilon - 0 = \epsilon \tag{9.22}$$

This result shows that ϵ is itself a deviation variable.

Since $\epsilon_s = 0$, Eq. (9.19) becomes at steady state

$$q_s = K_c\epsilon_s + A = 0 + A = A$$

Equation (9.19) may now be written in terms of q_s; *thus*

$$q = K_c\epsilon + q_s$$

or

$$Q = K_c\epsilon \tag{9.23}$$

where $Q = q - q_s$

The transform of Eq. (9.23) is simply

$$Q(s) = K_c\epsilon(s) \tag{9.24}$$

*In a practical situation, the equality among the three variables, T, T_m, and T_R, at steady state as given by Eq. (9.20) can always be established by adjustment of the instruments. The equality between T and T_m can be achieved by calibration of the measuring element. The equality between T_m and T_R can be achieved by adjustment of the proportional controller.

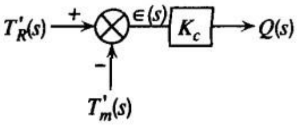

FIGURE 9-7
Block diagram of proportional controller.

Note that ϵ, which is also equal to ϵ', may be expressed as

$$\epsilon = T_R - T_{R_s} - (T_m - T_{m_s}) \tag{9.25}$$

or

$$\epsilon = T'_R - T'_m \tag{9.26}$$

Equation (9.25) follows from the definition of ϵ and the fact that $T_{R_s} = T_{m_s}$. Taking the transform of Eq. (9.26) gives

$$E(S) = T'_R(s) - T'_m(s) \tag{9.27}$$

The transfer function for the proportional controller given by Eq. (9.24) and the generation of error given by Eq. (9.27) may be expressed by the block diagram shown in Fig. 9.7.

We have now completed the development of the separate blocks. If these