4.3 Design of Sections for Flexure (Part II)

This section covers the following topics

- Final Design for Type 2 Members

The steps for Type 1 members are explained in Section 4.2, Design of Sections for Flexure (Part I).

4.3.1 Final Design for Type 2 Members

For Type 2 members, the tensile stress under service loads is within the cracking stress of concrete. The allowable tensile stress in concrete ($f_{ct,all}$) as per **IS:1343 - 1980** is same for transfer and service load conditions. The value is 3.0 N/mm^2, which can be increased to 4.5 N/mm^2 for temporary loads.

The following material provides the steps for sections with small self-weight moment. For sections with large self- weight moment, the eccentricity *e* may need to be determined based on the cover requirements.

1) Calculate eccentricity *e* to locate the centroid of the prestressing steel (CGS).

Under the self-weight, *C* may lie outside the kern region. The lowest possible location of *C* due to self-weight is determined by the allowable tensile stress at the top.

The following sketch explains the extreme location of *C* due to self-weight moment (M_{sw}) at transfer.

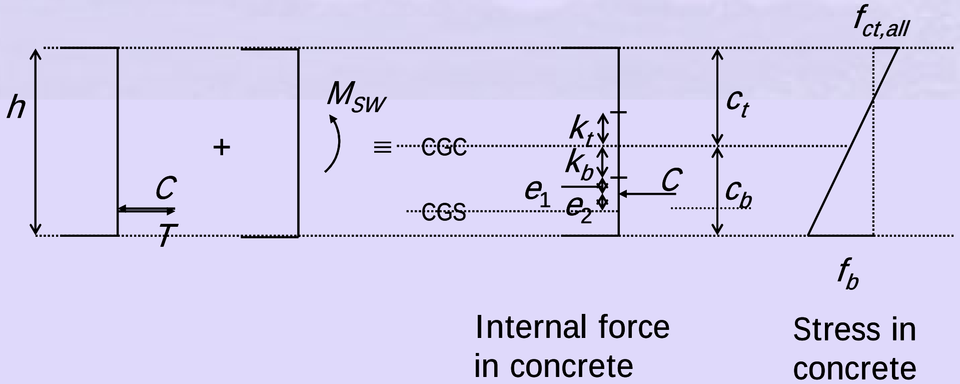


Figure 4-3.1 Stress in concrete due to compression outside bottom kern point

In the above sketch,

f_b = maximum compressive stress in concrete at bottom edge

$f_{ct,all}$ = allowable tensile stress in concrete at top edge

h = total height of the section

k_t, k_b = distances of upper and lower kern points, respectively, from CGC

c_t, c_b = distances of upper and lower edges, respectively, from CGC

e_1 = distance between the bottom kern point and the location of compression

e_2 = distance by which the compression travels from CGS due to self weight

P_0 = prestress at transfer after initial losses.

From the previous figure, the shift of C due to self-weight gives an expression of e_2. It is evident that if C is further shifted upwards by a distance e_1 to the bottom kern point, there will be no tensile stress at the top.

$$e_2 = \frac{M_{sw}}{P_0} \qquad \textbf{(4-3.1)}$$

The value of e_1 is calculated from the expression of stress corresponding to the moment due to the shift in C by e_1.

$$\frac{P_0 e_1 c_t}{I} = f_{ct,all}$$

$$e_1 = \frac{f_{ct,all} I}{P_0 c_t} \qquad \textbf{(4-3.2)}$$

$$e_1 = \frac{f_{ct,all} A k_b}{P_0}$$

Substituting $I = Ar^2$ and $r^2/c_t = k_b$

The distance of the CGS below the bottom kern point is given as follows.

$$e_1 + e_2 = \frac{M_{sw} + f_{ct,all} A k_b}{P_0} \qquad \textbf{(4-3.3)}$$

The eccentricity e is calculated from the following equation.

$$e = e_1 + e_2 + k_b$$

$$e = \frac{M_{sw} + f_{ct,all} A k_b}{P_0} + k_b \qquad \textbf{(4-3.4)}$$

The above expression can be compared with the expression of Type 1 member $e = (M_{sw} / P_0) + k_b$. Note that the eccentricity has increased for a Type 2 member due to the allowable tensile stress $f_{ct,all}$.

2) Recompute the effective prestress P_e and the area of prestressing steel A_p.
Under the total load, C may lie outside the kern region. The highest permissible location of C due to total load is determined by the allowable tensile stress at the bottom.

The following sketch explains the highest possible location of C due to the total moment (M_T).

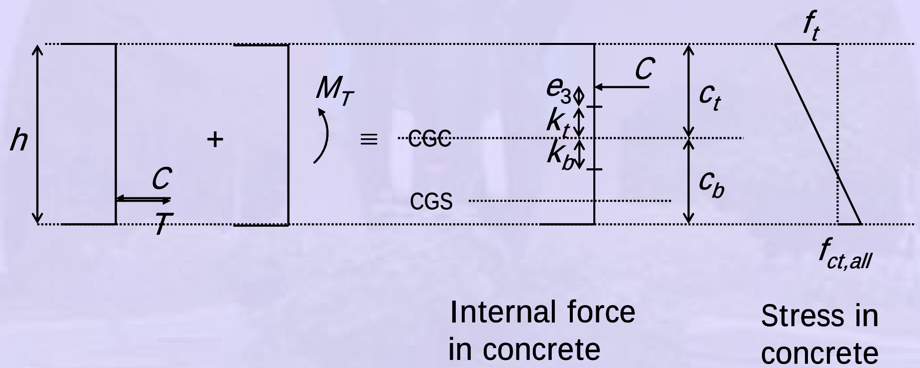


Figure 4-3.2 Stress in concrete due to compression outside top kern point

From the previous figure, the expression of e_3 is obtained by the tensile stress generated due to the shift of C beyond the upper kern point.

$$\frac{P_e e_3 c_b}{I} = f_{ct,all}$$

$$e_3 = \frac{f_{ct,all} I}{P_e c_b}$$

$$e_3 = \frac{f_{ct,all} A k_t}{P_e} \qquad \textbf{(4-3.5)}$$

Substituting $I = Ar^2$ and $r^2/c_b = k_t$

The shift of *C* due to the total moment gives an expression of P_e.

$$M_T = P_e\left(e + k_t + e_3\right)$$
$$= P_e\left(e + k_t\right) + f_{ct,all} A k_t$$
$$\therefore P_e = \frac{M_T - f_{ct,all} A k_t}{e + k_t} \tag{4-3.6}$$

The above expression can be compared with the expression of Type 1 member $P_e = M_T /(e + k_t)$. Note that the prestressing force has decreased for a Type 2 member due to the allowable tensile stress $f_{ct,all}$. This will lead to a decrease in the area of prestressing steel (A_p). Considering $f_{pe} = 0.7 f_{pk}$, A_p is recomputed as follows.

$$A_p = P_e / f_{pe} \tag{4-3.7}$$

3) Recompute eccentricity *e*

First the value of P_0 is updated. The eccentricity *e* is recomputed with the updated value of P_0.

If the variation of *e* from the previous value is large, another cycle of computation of the prestressing variables can be undertaken.

4) Check the compressive stresses in concrete

The maximum compressive stress in concrete should be limited to the allowable values.

At transfer, the stress at the bottom should be limited to $f_{cc,all}$, where $f_{cc,all}$ is the allowable compressive stress in concrete at transfer (available from **Figure 8** of **IS:1343 - 1980**). At service, the stress at the top should be limited to $f_{cc,all}$, where $f_{cc,all}$ is the allowable compressive stress in concrete under service loads (available from **Figure 7** of **IS:1343 - 1980**).

a) At Transfer

The stress at the bottom can be calculated from the stress diagram.

$$f_b = -\frac{C}{A} - \frac{C\left(k_b + e_1\right)c_b}{I}$$
$$f_b = -\frac{C}{A}\left(1 + \frac{k_b c_b}{r^2}\right) - \frac{C e_1 c_b}{I} \tag{4-3.8}$$

From $f_{ct,all} = Ce_1c_t / I$, substituting $Ce_1 / I = f_{ct,all} /c_t$

$$f_b = -\frac{C}{A}\left(1+\frac{c_b}{c_t}\right) - \frac{f_{ct,all}}{c_t}c_b$$

$$f_b = -\frac{C}{A}\frac{h}{c_t} - \frac{f_{ct,all}}{c_t}c_b \qquad \textbf{(4-3.9)}$$

To satisfy $|f_b| \le f_{cc,all}$, the area of the section (A) is checked as follows.

$$\frac{C}{A}\frac{h}{c_t} + \frac{f_{ct,all}}{c_t}c_b \le f_{cc,all}$$

$$\frac{P_0h}{A} \le f_{cc,all}c_t - f_{ct,all}c_b$$

$$\therefore \quad A \ge \frac{P_0h}{f_{cc,all}c_t - f_{ct,all}c_b} \qquad \textbf{(4-3.10)}$$

If A is not adequate then the section has to be redesigned.

b) At Service

The stress at the top can be calculated from the stress diagram.

$$f_t = -\frac{C}{A} - \frac{C(k_t + e_3)c_t}{I}$$

$$f_t = -\frac{C}{A}\left(1+\frac{k_tc_t}{r^2}\right) - \frac{Ce_3c_t}{I} \qquad \textbf{(4-3.11)}$$

From $f_{ct,all} = Ce_3c_b / I$, substituting $Ce_3 / I = f_{ct,all} /c_b$

$$f_t = -\frac{C}{A}\left(1+\frac{c_t}{c_b}\right) - \frac{f_{ct,all}c_t}{c_b}$$

$$f_t = -\frac{C}{A}\frac{h}{c_b} - \frac{f_{ct,all}c_t}{c_b} \qquad \textbf{(4-3.12)}$$

To satisfy $|f_t| \le f_{cc,all}$, the area of the section (A) is checked as follows.

$$\frac{C}{A}\frac{h}{c_b} + \frac{f_{ct,all}c_t}{c_b} \le f_{cc,all}$$

$$\frac{P_eh}{A} \le f_{cc,all}c_b - f_{ct,all}c_t$$

$$\therefore A \ge \frac{P_eh}{f_{cc,all}c_b - f_{ct,all}c_t} \qquad \textbf{(4-3.13)}$$

If A is not adequate then the section has to be redesigned.

The following table shows a comparison of equations for Type 1 and Type 2 members.

Table 4-3.1 Comparison of equations for Type 1 and Type 2 members

	Type 1	Type 2
Eccentricity	$e = \dfrac{M_{sw}}{P_0} + k_b$	$e = \dfrac{M_{sw} + f_{ct,all}Ak_b}{P_0} + k_b$
Effective prestress	$P_e = \dfrac{M_T}{e + k_t}$	$P_e = \dfrac{M_T - f_{ct,all}Ak_t}{e + k_t}$
Minimum area based on stress at bottom at transfer	$A \geq \dfrac{P_0 h}{f_{cc,all} c_t}$	$A \geq \dfrac{P_0 h}{f_{cc,all} c_t - f_{ct,all} c_b}$
Minimum area based on stress at top at service	$A \geq \dfrac{P_e h}{f_{cc,all} c_b}$	$A \geq \dfrac{P_e h}{f_{cc,all} c_b - f_{ct,all} c_t}$

The following example shows the design of a Type 2 prestressed member. The same section was designed as a Type 1 member in Section 4.2, Design of Sections for Flexure (Part I). The solutions of the two examples are compared at the end.

Example 4-3.1

Design a simply supported Type 2 prestressed beam with M_T = 435 kNm (including an estimated M_{SW} = 55 kNm). The height of the beam is restricted to 920 mm. The prestress at transfer f_{p0} = 1035 N/mm^2 and the prestress at service f_{pe} = 860 N/mm^2.

Based on the grade of concrete, the allowable compressive stresses are 12.5 N/mm^2 at transfer and 11.0 N/mm^2 at service. The allowable tensile stresses are 2.1 N/mm^2 at transfer and 1.6 N/mm^2 at service.

The properties of the prestressing strands are given below.

Type of prestressing tendon : 7-wire strand

Nominal diameter = 12.8 mm

Nominal area = 99.3 mm^2

Solution

A) Preliminary design

The values of h and M_{SW} are given.

1) Estimate lever arm z.

$$\frac{M_{sw}}{M_T} = \frac{55}{435}$$
$$= 12.5\ \%$$

Since $M_{SW} < 0.3\ M_T$,

use $z = 0.5h$

$= 0.5 \times 920$

$= 460$ mm

2) Estimate the effective prestress.

Moment due to imposed loads

$$\begin{aligned} M_{IL} &= M_T - M_{sw} \\ &= 435 - 55 \\ &= 380\ \text{kNm} \end{aligned}$$

Effective prestress

$$\begin{aligned} P_e &= \frac{380 \times 10^3}{460} \\ &= 826\ \text{kN} \end{aligned}$$

3) Estimate the area of the prestressing steel.

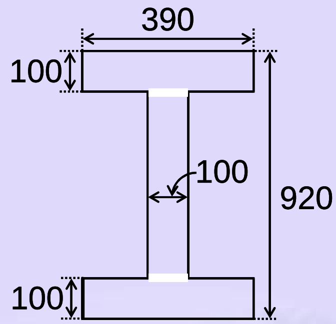
$$\begin{aligned} A_p &= \frac{P_e}{f_{pe}} \\ &= \frac{826 \times 10^3}{860} \\ &= 960\ \text{mm}^2 \end{aligned}$$

4) Estimate the area of the section to have average stress in concrete equal to 0.5 $f_{cc,all}$.

$$\begin{aligned} A &= \frac{P_e}{0.5 f_{cc,all}} \\ &= \frac{826 \times 10^3}{0.5 \times 11.0} \\ &= 150 \times 10^3\ \text{mm}^2 \end{aligned}$$

The following trial section has the required depth and area.

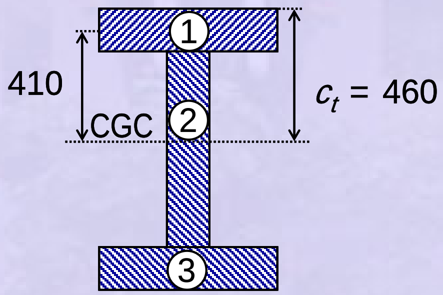
Trial cross-section



Values in mm.

B) Calculation of geometric properties

The section is symmetric about the horizontal axis. Hence, the CGC lies at mid depth. The section is divided into three rectangles for the computation of the geometric properties.



Values in mm.

Check area of the section

$$A = 2 \times (390 \times 100) + (720 \times 100)$$
$$= 150{,}000 \text{ mm}^2$$

Moment of inertia of the section about axis through CGC

$$I = 2I_1 + I_2$$
$$= 2\left[\frac{1}{12} \times 390 \times 100^3 + (390 \times 100) \times 410^2\right] + \frac{1}{12} \times 100 \times 720^3$$
$$= 1.6287 \times 10^{10} \text{ mm}^4$$

Square of the radius of gyration

$$r^2 = \frac{I}{A}$$
$$= \frac{1.6287\times10^{10}}{150{,}000}$$
$$= 108{,}580\ \text{mm}^2$$

Kern levels of the section

$$k_t = k_b = \frac{r^2}{c_t}$$
$$= \frac{108{,}580}{460}$$
$$= 236\ \text{mm}$$

Summary after preliminary design

Properties of section

A = 150,000 mm^2

I = 1.6287×1010 mm^4

c_t = c$_b$ = 460 mm

k_t = k_b = 236 mm

Values of prestressing variables

A_p = 960 mm^2

P_e = 826 kN

C) Final design

1) Calculate eccentricity e

$$P_0 = A_p f_{p0}$$
$$= 960\times1035$$
$$= 993.6\ \text{kN}$$

$$e_1 + e_2 = \frac{M_{sw} + f_{ct,all}Ak_b}{P_0}$$
$$= \frac{55\times10^3 + \frac{2.1}{10^3}\times150{,}000\times236}{993.6}$$
$$= 130\,\text{mm}$$

$$
\begin{aligned}
e &= e_1 + e_2 + k_b \\
&= 130 + 236 \\
&= 366 \text{ mm}
\end{aligned}
$$

2) Recompute the effective prestress and the area of prestressing steel A_p.

$$
\begin{aligned}
P_e &= \frac{M_T - f_{ct,all} A k_t}{e + k_t} \\
&= \frac{435 \times 10^3 - \dfrac{1.65}{10^3} \times 150{,}000 \times 236}{336 + 236} \\
&= 625.6 \text{ kN}
\end{aligned}
$$

Since P_e is substantially lower than the previous estimate of 826 kN, A_p, P_0 and e need to be recalculated.

$$
\begin{aligned}
A_p &= \frac{P_e}{f_{pe}} \\
&= \frac{625.6 \times 10^3}{860} \\
&= 727 \text{ mm}^2
\end{aligned}
$$

3) Recompute eccentricity e

$$
\begin{aligned}
P_0 &= A_p f_{p0} \\
&= 727 \times 1035 \\
&= 752.4 \text{kN}
\end{aligned}
$$

$$
\begin{aligned}
e &= \frac{M_{sw} + f_{ct,all} A k_b}{P_0} + k_b \\
&= \frac{55 \times 10^3 + \dfrac{2.1}{10^3} \times 150{,}000 \times 236}{752.4} + 236 \\
&= 172 + 236 \\
&= 408 \rightarrow 400 \text{ mm}
\end{aligned}
$$

Check the cover requirement

Assuming the outer diameter of duct equal to 54 mm

Clear cover for the duct $= 460 - 400 - \dfrac{1}{2} \times 54$

$= 33 \text{ mm}$

The clear cover at the bottom is greater than 30 mm (**Clause 11.1.6.2, IS: 1343 - 1980**), which is satisfactory. The side cover in the web is slightly less than 30 mm. The thickness of the web can be increased to satisfy the requirement.

Since the value of *e* has changed from 366 mm to 400 mm, prestressing variables are recomputed.

$$\begin{aligned} P_e &= \frac{M_T - f_{ct,all} A k_t}{e + k_t} \\ &= \frac{435\times10^3 - \dfrac{1.65}{10^3}\times150{,}000\times236}{400+236} \\ &= 592.0\ \text{kN} \end{aligned}$$

P_e has further reduced from 625.6 kN. A_p and P_0 are recalculated.

$$\begin{aligned} A_p &= \frac{592\times10^3}{860} \\ &= 688.5\ \text{mm}^2 \end{aligned}$$

Select (7) 7-wire strands with

$$\begin{aligned} A_p &= 7\times99.3 \\ &= 695.1\text{mm}^2 \end{aligned}$$

The tendons can be placed in one duct. The outer diameter of the duct is 54 mm.

$$\begin{aligned} P_0 &= 695.1\times1035 \\ &= 719.4\text{kN} \end{aligned}$$

Since the maximum possible eccentricity is based on cover requirement, the value of *e* is not updated.

4) Check the compressive stresses in concrete.

At transfer

$$\begin{aligned} A &\ge \frac{P_0 h}{f_{cc,all} c_t - f_{ct,all} c_b} \\ &= \frac{719.4\times10^3\times920}{12.5\times460 - 2.1\times460} \\ &= 138{,}352\ \text{mm}^2 \end{aligned}$$

At service

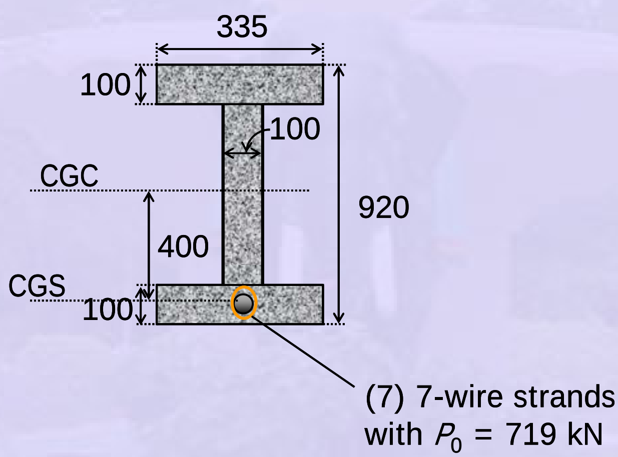
$$A \geq \frac{P_e h}{f_{cc,all} c_b - f_{ct,all} c_t}$$

$$= \frac{592 \times 10^3 \times 920}{11 \times 460 - 1.65 \times 460}$$

$$= 126{,}631 \text{mm}^2$$

The governing value of A is 138,352 mm^2. The section can be revised. The width of the flange is reduced to 335 mm. The area of the revised section is 139,000 mm^2. Another set of calculations can be done to calculate the geometric properties precisely.

Design cross-section at mid-span



Comparison of Type 1 and Type 2 sections

The solutions from the examples of Type 1 and Type 2 members are placed together in the next figure for comparison.

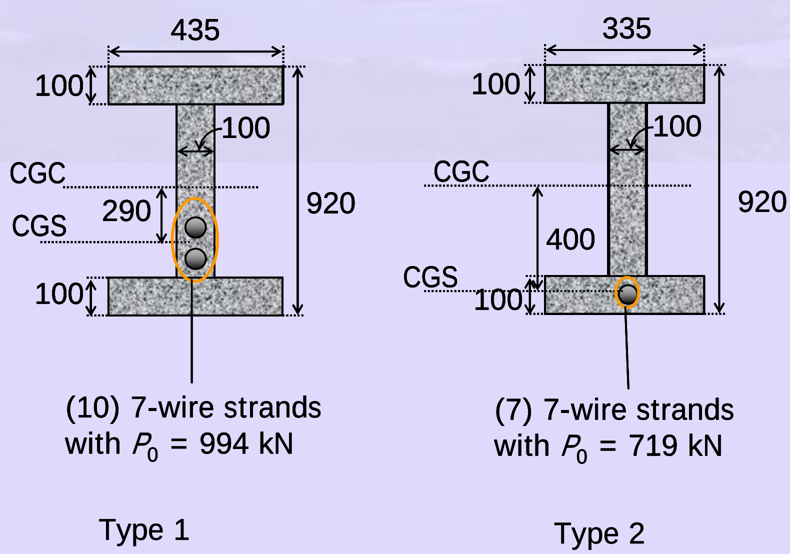


Figure 4-3.3 Sections designed as Type 1 and Type 2 members

The following observations can be made.

1) In Type 2 section, the amount of prestressing steel and the prestressing force are less than those in a Type 1 section. The area of cross-section is less for Type 2 section.

$\Rightarrow$ Type 2 section is relatively **economical**.

2) The eccentricity in Type 2 section is larger than in Type 1 section. For unit prestressing force, the prestressing is more **effective** in Type 2 section.