

3.6 Analysis of Members under Flexure (Part V)

This section covers the following topics.

- Analysis of Partially Prestressed Section
- Analysis of Unbonded Post-tensioned Beam
- Analysis of Behaviour

3.6.1 Analysis of Partially Prestressed Section

Introduction

The analyses that are presented in the earlier sections, are for members which do not have any conventional non-prestressed reinforcement. Usually conventional reinforcement is provided in addition to the prestressing steel. When this reinforcement is considered in the flexural capacity, the section is termed as a partially prestressed section.

The reasons for using a partially prestressed section are as follows.

- 1) The section is economical.
- 2) The cambering is less compared to an equivalent section without conventional reinforcement.
- 3) The ductility is more in a partially prestressed section.
- 4) Any reversal of moments (for example, due to earthquake) is not detrimental as compared to an equivalent section without conventional reinforcement.

Analysis

A partially prestressed section can be either rectangular or flanged. A section can be doubly reinforced with reinforcement near the compression face.

Here, the equations for a doubly reinforced rectangular section are given. The following sketch shows the beam cross section, strain profile, stress diagram and force couples at the ultimate state.

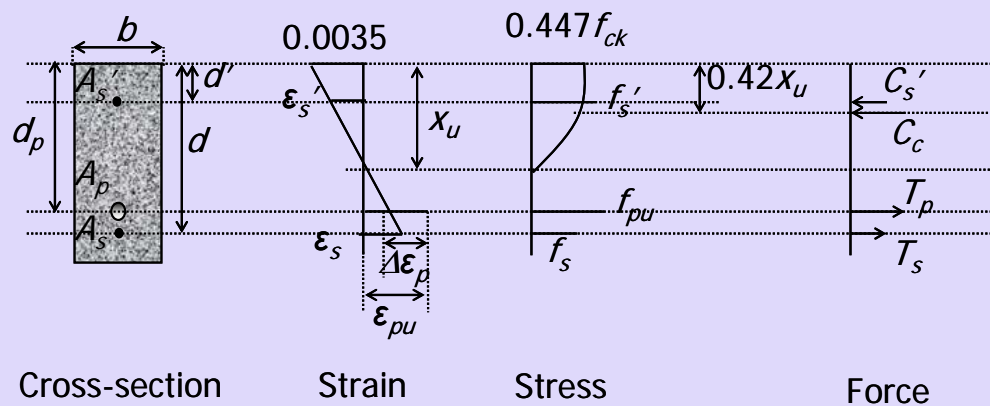


Figure 3-6.1 Sketches for analysis of a partially prestressed section

The variables in the above figure are explained.

- b = breadth of the section
- d = depth of the centroid of the reinforcing steel (tension side)
- d' = depth of the centroid of the reinforcing steel (compression side)
- d_p = depth of the centroid of prestressing steel (CGS)
- A_s = area of the reinforcing steel (tension side)
- A_s' = area of the reinforcing steel (compression side)
- A_p = area of the prestressing steel
- $\Delta\epsilon_p$ = strain difference in the prestressing steel when strain in concrete is zero
- x_u = depth of the neutral axis at ultimate
- ϵ_s = strain in reinforcing steel (tension side) at ultimate
- ϵ_s' = strain in reinforcing steel (compression side) at ultimate
- ϵ_{pu} = strain in prestressing steel at the level of CGS at ultimate
- f_s = stress in reinforcing steel (tension side) at ultimate
- f_s' = stress in reinforcing steel (compression side) at ultimate
- f_{pu} = stress in prestressing steel at ultimate.

The strain difference ($\Delta\epsilon_p$) is further explained in Section 3.4, Analysis of Member under Flexure (Part III).

The stress block in concrete is derived from the constitutive relationship for concrete. The relationship is explained in Section 1.6, Concrete (Part II). The compressive force in concrete can be calculated by integrating the stress block along the depth. The stress in the tendon is calculated from the constitutive relationship for prestressing steel. The relationship is explained in Section 1.7, Prestressing Steel.

The expressions of the forces are as follows.

$$C_s' = A_s' f_s' \quad (3-6.1)$$

$$C_c = 0.36 f_{ck} x_u b \quad (3-6.2)$$

$$T_p = A_p f_{pu} \quad (3-6.3)$$

$$T_s = A_s f_s \quad (3-6.4)$$

The strengths of the materials are denoted by the following symbols.

f_{ck} = characteristic compressive strength of concrete

f_{pk} = characteristic tensile strength of prestressing steel

f_y = characteristic yield stress of reinforcing steel

Based on the principles of mechanics (as explained under the Analysis of a Rectangular Section in Section 3.4, Analysis of Member Under Flexure (Part III)), the following equations are derived.

1) Equations of equilibrium

The first equation states that the resultant axial force is zero. This means that the compression and the tension in the force couple balance each other.

$$\begin{aligned} \sum F &= 0 \\ \Rightarrow T_u &= C_u \\ \Rightarrow T_p + T_s &= C_c + C_s' \\ \Rightarrow A_p f_{pu} + A_s f_s &= 0.36 f_{ck} x_u b + A_s' f_s' \end{aligned} \quad (3-6.5)$$

The second equation relates the ultimate moment capacity (M_{uR}) with the internal couple in the force diagram.

$$\begin{aligned} M_{uR}|_{A_s} &= T_s (d - d_p) + C_c (d_p - 0.42 x_u) + C_s' (d_p - d') \\ &= A_s f_s (d - d_p) + 0.36 f_{ck} x_u b (d_p - 0.42 x_u) + A_s' f_s' (d_p - d') \end{aligned} \quad (3-6.6)$$

2) Equations of compatibility

For each layer of steel there is a compatibility equation. If there are distributed reinforcing bars in several layers and the spacing between the layers is large, then the use of compatibility equation for each layer is more accurate than the use of one compatibility equation for the centroid of the layers. The following equations are developed based on the similarity of the triangles in the strain diagram.

$$\frac{x_u}{d_p} = \frac{0.0035}{0.0035 + \varepsilon_{pu} - \Delta\varepsilon_p} \quad (3-6.7)$$

$$\frac{d - x_u}{x_u} = \frac{\varepsilon_s}{0.0035} \quad (3-6.8)$$

$$\frac{x_u - d'}{x_u} = \frac{\varepsilon_s'}{0.0035} \quad (3-6.9)$$

3) Constitutive relationships

a) Concrete

The constitutive relationship for concrete is considered in the expressions of C_c . This is based on the area under the design stress-strain curve for concrete under compression.

b) Prestressing steel

$$f_{pu} = F_1(\varepsilon_{pu}) \quad (3-6.10)$$

c) Reinforcing steel

$$f_s = F_2(\varepsilon_s) \quad (3-6.11)$$

$$f_s' = F_3(\varepsilon_s') \quad (3-6.12)$$

For mild steel

$$f_s' = 0.87f_y \quad (3-6.13)$$

$$f_s = 0.87f_y \quad (3-6.14)$$

The known variables in an analysis are: b , d , d' , d_p , A_s , A_s' , A_p , $\Delta\varepsilon_p$, f_{ck} , f_y and f_{pk} .

The unknown quantities are: M_{uR} , x_u , ε_s , ε_s' , ε_{pu} , f_s , f_s' and f_{pu} .

The objective of the analysis is to find out M_{uR} , the ultimate moment capacity. The previous equations can be solved by the **strain compatibility method** as discussed for the fully prestressed rectangular section.

- 1) Assume x_u .
- 2) Calculate ε_{pu} from Eqn. 3-6.7.
- 3) Calculate f_{pu} from Eqn. 3-6.10.
- 4) Calculate T_p from Eqn. 3-6.3.
- 5) Calculate ε_s from Eqn. 3-6.8.
- 6) Calculate f_s from Eqn. 3-6.11.
- 7) Calculate T_s from Eqn. 3-6.4.
- 8) Calculate ε_s' from Eqn. 3-6.9.

9) Calculate f_s' from Eqn. 3-6.12.

10) Calculate C_s' from Eqn. 3-6.1.

11) Calculate C_c from Eqn. 3-6.2.

If Eqn. 3-6.5 ($T_u = C_u$) is not satisfied, change x_u .

If $T_u < C_u$ decrease x_u . If $T_u > C_u$ increase x_u .

12) Calculate M_{uR} from Eqn. 3-6.6.

The capacity M_{uR} can be compared with the demand under ultimate loads.

3.6.2 Analysis of Unbonded Post-tensioned Beam

In an unbonded post-tensioned beam, the ducts are not grouted. Hence, there is no strain compatibility between the steel of the tendons and the concrete at a section. The compatibility is in terms of deformation over the length of the member.

A sectional analysis is not possible. The analysis involves integrating the strain in concrete to calculate the deformation over the length of the member. The equation of compatibility is given as follows.

$$\Delta_p = \Delta_{cp} \quad (3-6.15)$$

Here,

Δ_p = deformation of the tendon

Δ_{cp} = deformation of the concrete at the level of prestressing steel (CGS).

The change in stress in steel (Δf_p) at ultimate is determined from Δ_p . The stress in steel at ultimate is given by the sum of the effective prestress (f_{pe}) and Δf_p .

$$f_{pu} = f_{pe} + \Delta f_p \quad (3-6.16)$$

The value of f_{pu} is less than that for a bonded tendon. The ultimate moment is given by the following equation.

$$M_{uR} = A_p f_{pu} (d - 0.42x_u) \quad (3-6.17)$$

The rigorous method of evaluating f_{pu} , based on deformation compatibility, is difficult. **IS:1343 - 1980** allows to calculate f_{pu} and x_u approximately from Table 12, Appendix B, based on the amount of prestressing steel. The later is expressed as the reinforcement index $\omega_p = A_p f_{pk} / bdf_{ck}$. Table 12 is reproduced as Table 3-6.1 which is applicable for

unbonded post-tensioned beams. The values of f_{pu} and x_u are given as f_{pu}/f_{pe} and X_u/d , respectively. The effective prestress (after the losses) in a tendon is represented as f_{pe} .

Table 3-6.1 Values of x_u and f_{pu} for unbonded post-tensioned rectangular beams
(Table 12, IS:1343 - 1980)

ω_p	f_{pu}/f_{pe} For values of L/d			x_u/d For values of L/d		
	30	20	10	30	20	10
0.025	1.23	1.34	1.45	0.10	0.10	0.10
0.05	1.21	1.32	1.45	0.16	0.16	0.18
0.10	1.18	1.26	1.45	0.30	0.32	0.36
0.15	1.14	1.20	1.36	0.44	0.46	0.52
0.20	1.11	1.16	1.27	0.56	0.58	0.64

The values of f_{pu}/f_{pe} and x_u/d from Table 3-6.1 are plotted in Figures 3-6.2 and 3-6.3, respectively. It is observed that with increase in ω_p , f_{pu} reduces and x_u increases. This is expected because with increase in the amount and strength in the steel, the stress in steel drops and the depth of the neutral axis increases to maintain equilibrium.

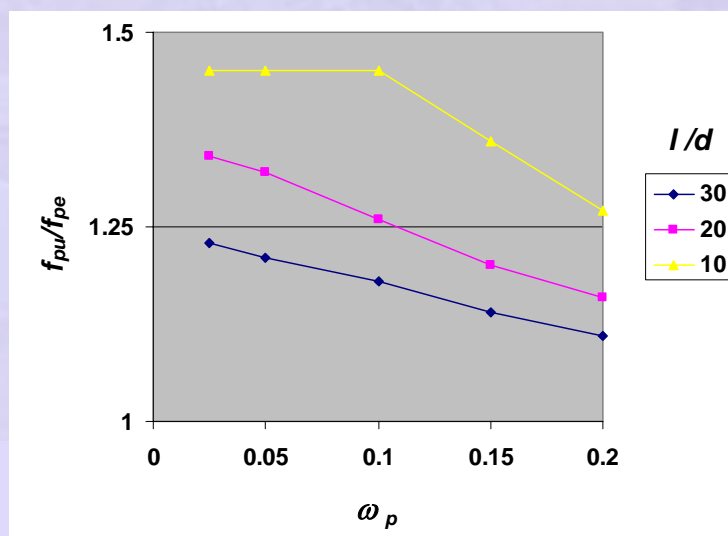


Figure 3-6.2 Variation of f_{pu} with respect to w_p (Table 3-6.1)

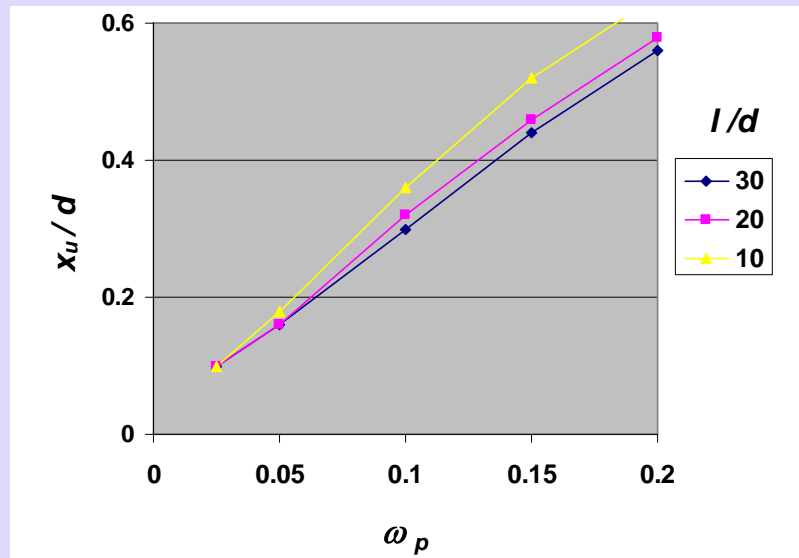


Figure 3-6.3 Variation of x_u/d with respect to w_p (Table 3-6.1)

Thus given the value of ω_p for a section, the values of f_{pu} and x_u can be approximately calculated from the above tables.

3.6.3 Analysis of Behaviour

The analysis of behaviour refers to the determination of the complete moment versus curvature behaviour of the section. The analyses at transfer, under service loads and for ultimate strength correspond to three instants in the above behaviour.

The curvature (φ) is defined as the gradient of the strain profile.

$$\varphi = \frac{\varepsilon_c + \varepsilon_{cp}}{d} \quad (3-6.18)$$

Here,

ε_c = extreme concrete compressive strain

ε_{cp} = strain in concrete at the level of prestressing steel (CGS)

d = depth of the CGS.

The following sketch shows the curvature (φ) in the strain profile.

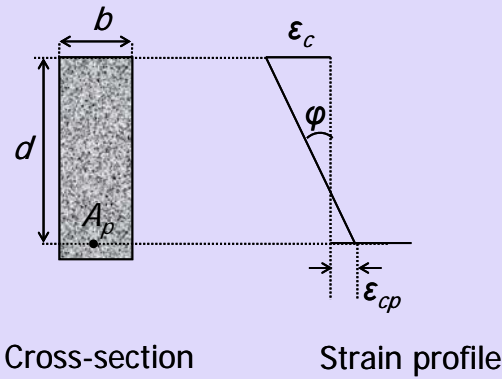


Figure 3-6.4 Definition of curvature

The analysis of behaviour involves the following three principles of mechanics.

- 1) **Equilibrium of internal forces** with the external loads at any point of the behaviour. There are two equilibrium equations.
 - a) Force equilibrium equation
 - b) Moment equilibrium equation.

The internal forces in concrete and steel are evaluated based on the respective strains, cross-sectional areas and the constitutive relationships.

- 2) **Compatibility of the strains** in concrete and in steel for bonded tendons. This assumes a perfect bond between the two materials. For unbonded tendons, the compatibility is in terms of deformation.
- 3) **Constitutive relationships** relating the stresses and the strains in the materials. The relationships are developed based on the material properties.

The equilibrium and compatibility equations and the constitutive relationships can be solved to develop the moment versus curvature curve for a section.

The following plot shows the curves for a prestressed section and a non-prestressed section. The two sections are equivalent in their ultimate flexural strengths.

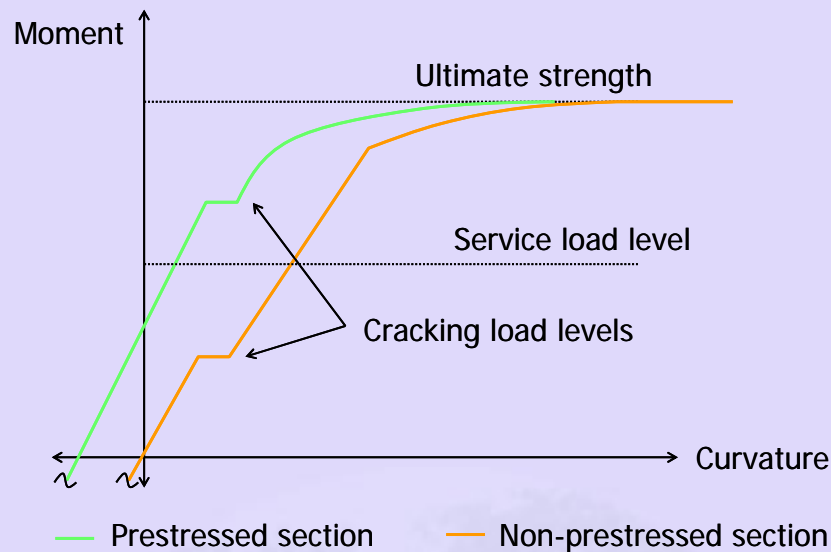


Figure 3-6.5 Moment versus curvature curves

From the previous plot, the following can be inferred.

- 1) Prestressing increases the cracking load. This leads to the following benefits.
 - Reduction of steel corrosion
⇒ Increase in durability.
 - Full section is utilised
⇒ Higher moment of inertia (higher stiffness)
⇒ Less deformations (improved serviceability).
 - Increase in shear capacity.
- 2) Prestressing shifts the curve from the origin.
 - For the prestressed member, there is a negative curvature causing camber in absence of external moment.
 - A certain amount of external moment is required to straighten the member.
- 3) For a given moment, the curvature of the prestressed member is smaller.
 - Prestressing reduces curvature at service loads.
- 4) For a given reverse moment, the curvature of the prestressed member is larger.
 - Prestressing is detrimental for the response under reverse moment.
- 5) The ultimate strength of the prestressed member is lower.
 - Prestressing is detrimental under reverse moment.
- 6) For a partially prestressed section with the same ultimate strength, the moment versus curvature curve will lie in between the curves for prestressed and non-prestressed sections.

Ductility

The ductility is a measure of energy absorption. For beams, the curvature ductility (μ) is defined as

$$\mu = \frac{\varphi_u}{\varphi_y} \quad (3-6.19)$$

Here,

φ_u = curvature at ultimate

φ_y = curvature at yield.

For prestressed beams, φ_y can be defined corresponding to a plastic strain of 0.002 in the prestressing tendons. It has been observed that the ductility of prestressed beams is less than that in reinforced concrete beams.

In design of members for seismic forces, ductility is an important requirement. In addition, seismic forces lead to reversal of moments near the supports of beams in a moment resisting frame. Hence, prestressing of beams in a moment resisting frame is not recommended in seismic areas.

Experimental Investigation

The behaviour of a beam and its ultimate strength can be determined by testing prototype specimens. The tests can be conducted under static or dynamic loads. Testing also helps to check the performance of the anchorage units. The following photo shows the set-up for testing a prototype bridge girder.



Figure 3-6.6 Set-up for testing a bridge girder